A REALIZATION OF WAVELET FILTER BANK WITH ADAPTIVE FILTER PARAMETERS

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ABSTRACT

In this paper, an efficient realization of the two-channel wavelet filter bank with adaptive filter parameters is proposed. Described time variant wavelet filter bank is more suitable for analysis of non-stationary signals then fixed banks. Basic convergence and regularity properties of the limit wavelet functions and scales are provided by fixed part of the filter bank. Variable part adapts to the analyzed signal. Proposed filter bank combines sub-band decomposition and parametric modeling. Realization is based on the lifting scheme, derived from a method of fixed wavelet filter bank design. Original Lagrange interpolation of samples in the time domain is modified to an approximation scheme that is recomputed at each step of decomposition. Adaptation criterion is calculated from wavelet coefficients, which is under some restrictions reproducible on the reconstruction side. Wavelet filter banks with adaptive filter parameters can outperform fixed banks in a number of applications.

1 INTRODUCTION

Analytical properties of wavelet filter banks are closely related to the convergence and regularity of limit wavelet functions and scales. More zero moments correspond to more regularity, which results in better description of smooth and correlated parts of the analyzed signal [1][2]. A number of well-known wavelet families has been developed, based on criteria such as orthogonality, minimum phase, symmetric impulse response, and many others. Number of vanishing moments of a fixed filter bank is usually chosen as a compromise between filter complexity and desired regularity. For a given order, wavelet filters usually have all zeros of their frequency response on Nyquist or DC frequency. But, it does not necessary result in maximum selectivity of the filter bank for a given input signal.

Our goal is to change filter parameters in one or both filters of the bank at each step of decomposition, depending on the analyzed signal. The two-channel PR filter bank should form wavelet tree or wavelet packet tree, so the convergence and some degree of regularity must remain. The adaptation criterion should be computed from wavelet coefficients, wishfully resulting in more compact representation of the analyzed signal. We expect benefits of using adaptive filter banks in many applications [6].

In section 2 we describe construction of the proposed adaptive filter bank. Sweldens 96 [3] proposed a construction of biorthogonal wavelet filter banks based on the lifting scheme, using interpolation of samples in the time domain. A short review of the "cakewalk" construction of wavelets is given in paragraph 2.1. Even samples are estimated from odds using Lagrange interpolation functions of chosen order. In the proposed scheme, we consider odd order polynomials corresponding to the even length FIR filters.

In paragraph 2.2 we give the proposed factorization of the adjustable lifting step. In section 3 we discuss the adaptation criterion, as well as the decomposition results. To ensure convergence and minimum regularity, filters are split in the fixed and the variable part. Filter parameters change at each step of decomposition, so the associated limit wavelet functions and scales change, too. We deal with a kind of "generalized" wavelets. Fixed vanishing moments plus limitations on variable filter parameters ensure convergence and reasonable regularity of decomposition functions.

2 FILTER BANK STRUCTURE

2.1 Lifting scheme

The lifting scheme is related to the polyphase representation of filter banks, with polyphase matrix factored in a cascade of triangular sub-matrices. Each sub-matrix corresponds to one lifting or dual lifting step. Its all-ones diagonal form guaranties existence of the inverse sub-matrix, even if lifting or dual lifting operators are not constant or linear. An inverse sub-matrix is obtained by a simple transposition followed by change in sign. It enables easy construction of perfect reconstruction time-variant and non-linear filter banks. Daubechies and Sweldens 98 [4] show that any two-band FIR filter bank can be factored in a set of lifting steps, using Euclidean algorithm.

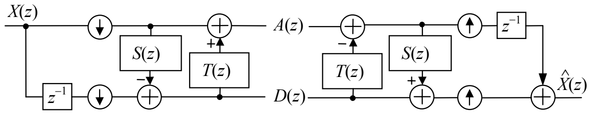

Figure 1. Two-channel PR filter bank factored in lifting and dual lifting steps.

The polyphase matrix of the filter bank from **Figure 1** is factored in 2 triangular sub-matrices:

$$\mathbf{P}(z) = \begin{bmatrix} 1 & 0 \\ T(z) & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -S(z) \\ 0 & 1 \end{bmatrix}.$$

$$H(z) = z^{-1} - 1 \cdot S(z^2),$$

$$L(z) = 1 + H(z) \cdot T(z^2).$$

We limit to FIR lifting steps $S(z)$ and $T(z)$, with $2N$ taps:

$$S(z) = s_0 z^{N-1} + \ldots + s_{N-1} + s_N z^{-1} + \ldots + s_{2N-1} z^{-N},$$

$$T(z) = t_0 z^{N} + ... + t_{N-1} z + t_N + ... + t_{2N-1} z^{-N+1}.$$

In this paper, a class of two-channel biorthogonal and time variant PR filter banks is constructed by the lifting scheme. Sweldens 96 [3] described a lifting scheme design of Deslauriers - Dubuc filter banks [5] by interpolation of samples in the time domain. The illustration of linear (II) and cubic (IV) case is given below:

$$d_{\mathrm{II}}[k] = x_o[k] - \frac{x_e[k-1] + x_e[k]}{2}$$

$$d_{\mathrm{IV}}[k] = x_o[k] - \frac{1}{16}(-x_e[k-2] + 9x_e[k-1] + 9x_e[k] - x_e[k+1])$$

$$S_{\mathrm{II}} = \frac{1}{2}(1 + z^{-1}) \qquad S_{\mathrm{IV}} = \frac{1}{16}(-z + 9 + 9z^{-1} - z^{-2})$$

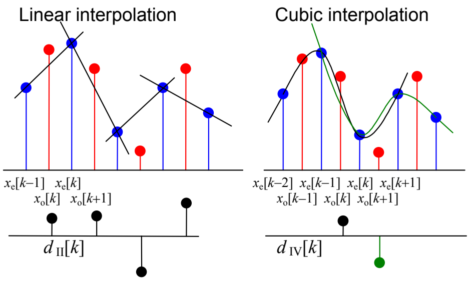

Figure 2. Lagrange interpolation of samples.

2.2 Adjustable lifting step

At first, we construct the predictor:

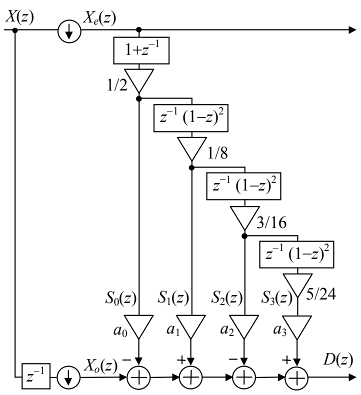

Figure 3. Realization of the lifting step.

Lifting step is realized as a weighted sum of additive components: $S_0(z)=a_0\cdot S_0(z)-a_1\cdot S_1(z)+a_2\cdot S_2(z)-a_3\cdot S_3(z)$, where:

$$S_0 = \frac{1}{2}(1 + z^{-1}), \qquad S_1 = -\frac{1}{16}(z - 1 - z^{-1} + z^{-2}),$$

$$S_2 = \frac{3}{256}(z^2 - 3z + 2 + 2z^{-1} - 3z^{-2} + z^{-3}),$$

$$S_3 = \frac{5}{2048}(z^3 - 5z^2 + 9z - 5 - 5z^{-1} + 9z^{-2} - 5z^{-3} + z^{-4}).$$

If the multiplying factors $\{a_0, a_1, a_2, a_3\}$ are constant, chosen from sets $\{1,0,0,0\}_{\mathrm{I}}$, $\{1,1,0,0\}_{\mathrm{II}}$, $\{1,1,1,0\}_{\mathrm{III}}$, $\{1,1,1,1\}_{\mathrm{IV}}$ we have lifting steps $S_{\mathrm{I}}(z)=S_0(z)$, $S_{\mathrm{II}}(z)=S_0(z)-S_1(z)$, $S_{\mathrm{III}}(z)=S_0(z)-S_1(z)+S_2(z)$, and $S_{\mathrm{IV}}(z)=S_0(z)-S_1(z)+S_2(z)-S_3(z)$. Resulting S_{I}–S_{IV} correspond to prediction of odd samples from neighboring even samples using linear, cubic, 5th order or 7th order Lagrange interpolation, as illustrated in **Figure 2**. Also, they correspond to 2, 4, 6 or 8 zero moments of the associated wavelet function. This scheme can be easily extended to more vanishing moments, if needed.

Now, we split the proposed structure in the fixed and variable part. We achieve desired number of vanishing moments by fixing factors from a_0 to a_3. For example, $a_0=1$ results in 2, or $a_0=a_1=1$ results in 4 vanishing moments (linear or cubic interpolation). Then, we use the residual multipliers as parameters that could be changed at each step of decomposition.

It leads to a modified approximation scheme, where prediction polynomials are set to minimize error signal derived from $d[k]$, at each moment k, on chosen adaptation interval $[k-K_1, k+K_2]$.

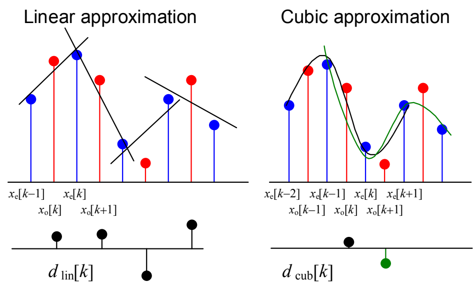

Figure 4. Modified scheme. Approximation polynomials do not necessarily contain known points. They are rather set to minimize some property of the signal $d[k]$ (e.g., the energy of d in some neighborhood of the observed step k).

2.3 Adjustable dual lifting step

Now, we construct the update step:

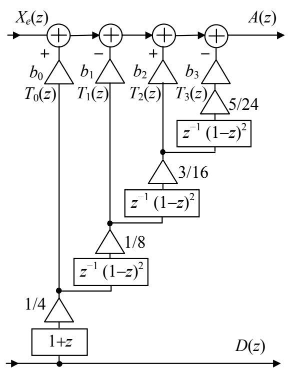

Figure 5. Realization of the dual lifting step

For the moment, let us assume that the lifting step multipliers $\{a_i\}$ are chosen from sets I-IV (ones or zeros). In order to provide vanishing moments to the scale function, we set the gains b_i:

$\{a_i\} \rightarrow$	I	II	III	IV
b_0	**1**	**1**	**1**	**1**
b_1	3/2	**1**	**1**	**1**
b_2	5/3	3/2	**1**	**1**
b_3	7/4	3/2	3/2	**1**

Table 2.1. Gain b_i depends on the actual number of zeros of the high-pass filter, unless we fix less or equal number of zeros of the low-pass filter.

An interesting conclusion comes from **Table 2.1**. If the number of zeros of the LP filter (f=Nyquist) is less or equal to the number of zeros of the HP filter (f=0), gains b_i equal 1 for all i=0–3. Hence, if the number of fixed gains a_i does not exceed the number of fixed gains b_i – we have "independent" vanishing moments. Moreover, they do not depend on the remaining free parameters. If we need more zeros for the LP filter, we can simply swap the position of HP and LP filter by reversing signs of all additive components S_i and T_i.

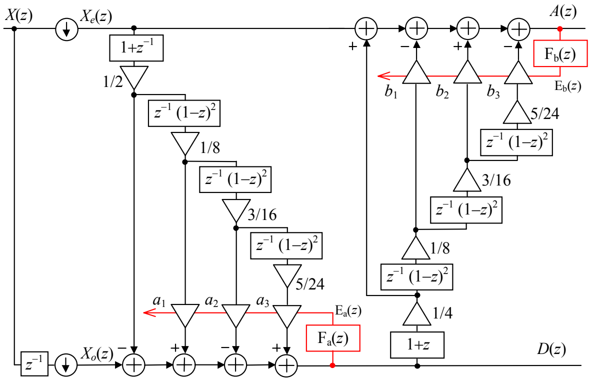

Figure 6. Adaptive wavelet filter bank (analysis part) with 2+2 vanishing moments and 3+3 free parameters.

3 ADAPTATION CRITERION AND RESULTS

We give an example with 2 vanishing moments fixed on both filters. Hence, gains a_0 and b_0 equal 1, and our filter bank is split in basic HP and LP channels. For the simplicity, we change only one free parameter: b_1, while all others are set to zero. Zero locus plot of the LP filter is shown in the following figure:

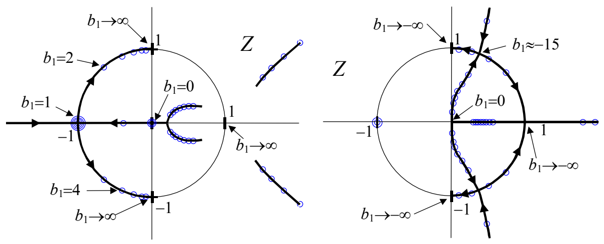

Figure 7. Zero locus plots of the LP filter, parameter b_1.

Positive values of b_1 are good candidates for achieving desired LP frequency response: zeros on the higher half of the unit circle can be adjusted to cancel large high frequency components of the analyzed signal. On the other hand, large negative values place zeros on the lower half of the circle, which is in conflict with convergence condition (Mallat, [7]). Both large positive or negative values widen the energetic frame bounds of the transform, thus making filter bank very far from unitary (more on frames in [2]). In practice, the region of acceptable parameter values is limited. Limit scale functions for several values of parameter b_1 are shown in **Figure 8**.

Due to decimation, aliasing frequency of the analyzed signal $x[k]$ maps to the DC component of the wavelet coefficients $d[k]$. Signal DC is preserved in $a[k]$ coefficients. To avoid its influence on the criterion, we use the high-pass filtered wavelet coefficients as the error input of the windowed least squares adaptation algorithm (illustration in **Figure 6**).

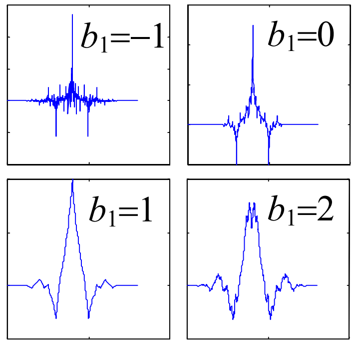

Figure 8. Limit scale functions for different constant values of parameter b_1. Top left: b_1=–1. Top right: b_1=0. Bottom left: b_1=1 (4 zero moments). Bottom right: b_1=2.

We applied the adaptive wavelet filter bank to a synthetic signal $x[k]$ composed from 3 sine waves of different frequencies.

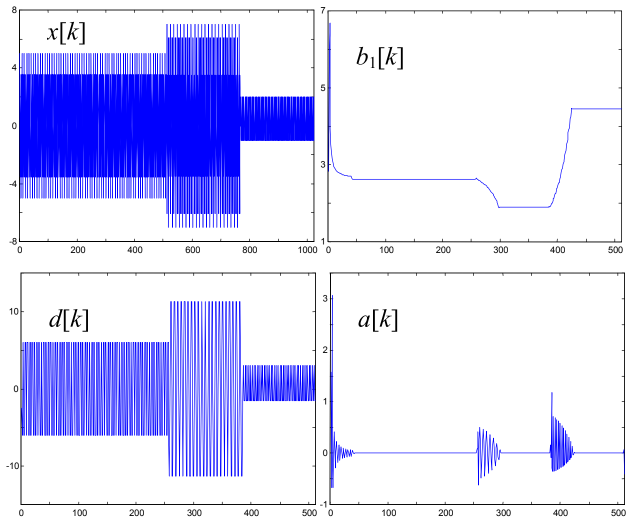

Figure 9. Top left: analyzed signal $x[k]$ composed from 3 sine waves ($\omega = 9\pi/12,\ 10\pi/12,\ 8\pi/12$). Top right: filter parameter $b_1[k]$ adapted by the least squares algorithm on interval [k–20, k+20]. Bottom right: detail $d[k]$. Bottom right: residual approximation coefficients $a[k]$.

The decomposition is almost optimal: filter parameter b_1 chosen by the adaptation criterion follows the frequency of the analyzed signal, while approximation coefficients are turned to zero after the adaptation. Scale functions change at each step of decomposition, somewhere "in between" of those shown in **Figure 8**, so the good regularity properties remain.

Figure 10 shows the analysis of the signal $x[k]$ from **Figure 9** with "skyline" component added. Periodic components of the analyzed signal are almost cleared from the approximation coefficients, after the adaptation of parameter b_1. The skyline component brought some variance to $b_1[k]$, when compared to decomposition in **Figure 9**. Low-pass filtering of the filter parameter b_1 gives slightly better decomposition results, which is shown in the left of **Figure 13**.

If we use real world signals, the unpredictable noise components may excel the correlated parts. The consequence is an intensive variance of the filters' parameters. Additional averaging of the filter parameters with $F(z)=1/10\cdot(z^5+z^4+...+1+...+z^{-3}+z^{-4})$ reduces the variance and enhances the decomposition properties (the right of **Figure 13**).

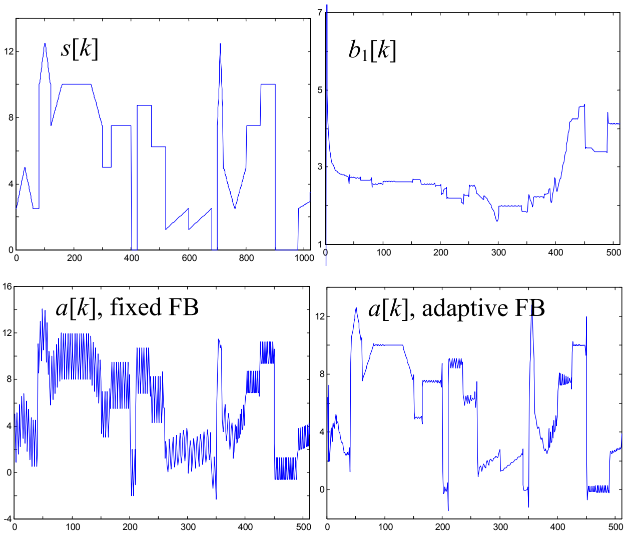

Figure 10. Upper left: “skyline” signal added to the signal $x[k]$ from **Figure 9**. Upper right: filter parameter $b_1[k]$, adapted by the filtered least squares algorithm on interval $[k–20, k+20]$. Lower left: approximation coefficients $d[k]$ computed using fixed wavelets with 2 zero moments (2,2). Lower right: approximation coefficients $a[k]$ computed using adaptive wavelets $(2, 2+b_1)$.

In the next example, we fix 2 vanishing moments on both filters ($a_0 = b_0 = 1$) and change one free parameter in the lifting step: a_1. All other parameters are set to zero. We analyze a short voice signal composed from a consonant followed by a vowel.

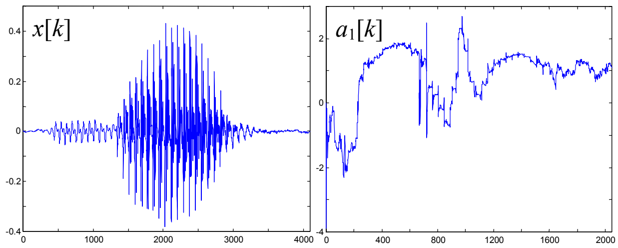

Figure 11. Left: voice signal $x[k]$. Right: filter parameters $a_1[k]$ adapted by the least squares algorithm on interval $[k–50, k+50]$.

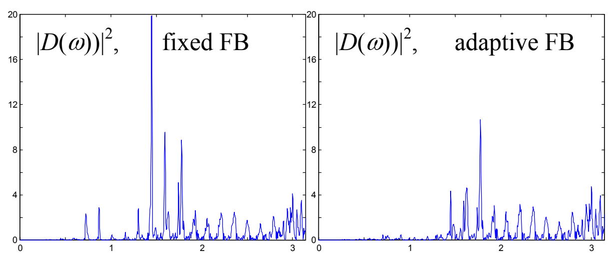

Figure 12. Spectral contents of detail coeficients $|D(\omega))|^2$. Left: fixed filter bank (2,2). Right: adaptive bank $(2+a_1, 2)$.

Adaptive FB reduces the highest frequency components of the detail coefficients, while keeping approximation coefficients almost unchanged (**Figure 12**).

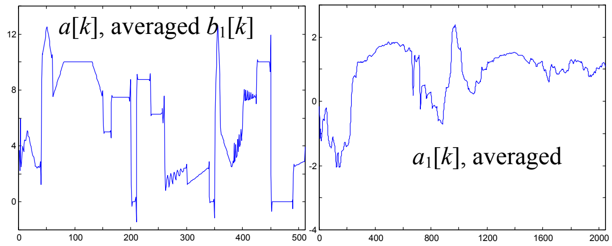

Figure 13. Left: Approximation coefficients from example in **Figure 10** computed by averaged filter parameters $b_1[k]$. Right: averaged filter parameters $a_1[k]$ from **Figure 11**

In general, we can reconstruct the analyzed signal from wavelet coefficients plus information on filter parameters $\{a_i\}$ and $\{b_i\}$. They can be coded very efficiently. But, if the adaptation criterion is causal, e.g. if current parameters are determined exclusively from **previous** wavelet coefficients, the adaptation algorithm can be self-reproduced on the reconstruction side. In that case, perfect reconstruction does not require filter parameters to be separately transferred to the reconstruction side.

4 Conclusion

We give an efficient realization of the two-channel adaptive wavelet filter bank. Prediction and update filters are implemented as a mixed cascade/parallel parallel set of filter sections, where each successive section brings the contribution of the higher order approximation. A set of filter parameters is fixed and determines the desired number of zero moments. Free parameters change at each step of decomposition or reconstruction. We used the least square error criterion, computed from filtered wavelet coefficients. Adaptive filter bank is applied on a synthetic signal. Wavelet coefficients get close to what we expect to be an optimal representation of the analyzed signal. Real world signals usually contain non-correlated components, inherent to the signal or caused by additive noise. They cause intensive variance of the free parameters, which can be handled by averaging. Described time variant wavelet filter bank is more suitable for analysis of non-stationary signals then fixed banks. It can outperform fixed wavelets in many applications.

References

[1] Daubechies I., “Orthonormal Bases of Compactly Supported Wavelets”, Comm. on Pure and Appl. Math., 41:909-996, 1988

[2] Daubechies I., Ten Lectures on Wavelets, Society for Industrial and Appl. Math., Capital City Press, Vermont 1992

[3] Sweldens W., “The lifting scheme: A custom-design construction of biorthogonal wavelets”, Appl. Comp. Harm. Anal., 3(2):186-200, 1996

[4] Daubechies I. and Sweldens W., “Factoring Wavelet Transforms into Lifting Steps”, J. Fourier Anal. and Appl., 4(3):247-269, 1998

[5] G. Deslauriers and S. Dubuc, “Interpolation dyadique”, In Fractals, dimensions non entiéres et applications, pp. 44-55, Masson, Paris, 1987

[6] Seršić D. and Lončarić S., “Enhancement of mammographic images for detection of microcalcifications”, Proceedings of the EUSIPCO 98, Rodos, Greece, 1998, pp. 4:2513-2516.

[7] Mallat S., “Multiresolution approx. and wavelet orthonormal bases on $L_2(R)$”, Trans. Am. Math. Soc. 315:69-87, 1989